Chapter (2)

Conduction

Goals:

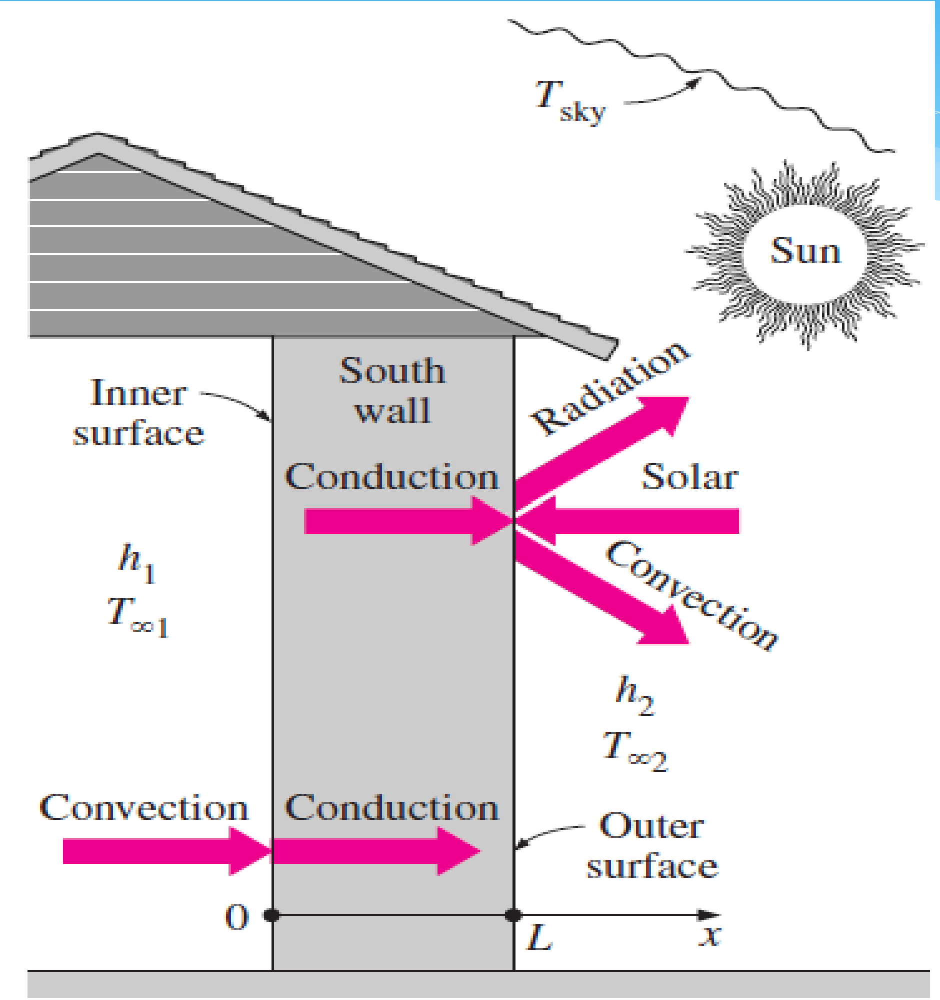
By the end of today's lecture, you should be able to:

- Learn how to deal with combined modes and composite walls for the three applications, plan wall, cylinder and sphere.

- You will be familiar with the concept of contact resistances and critical radios.

Combined Modes

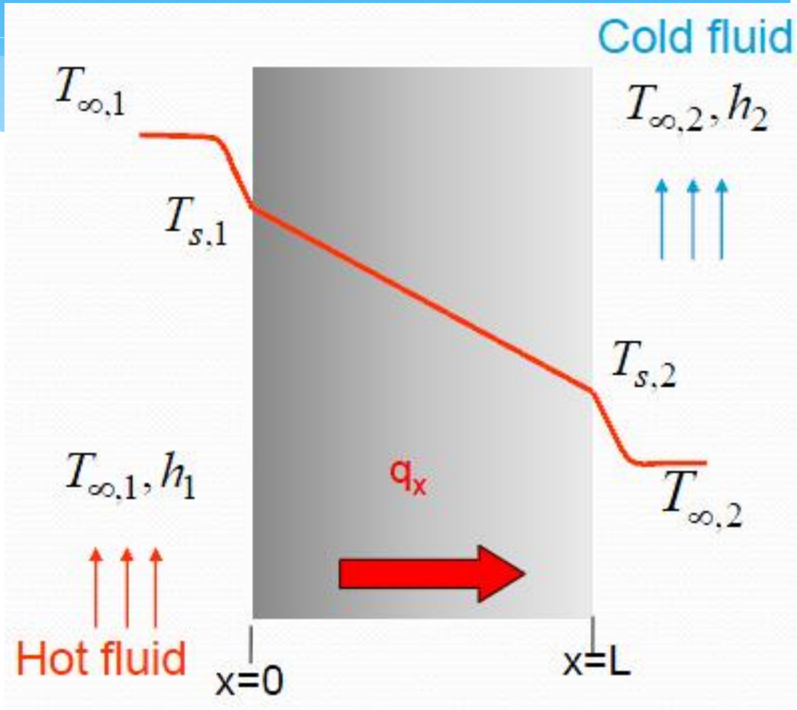
Combined Modes

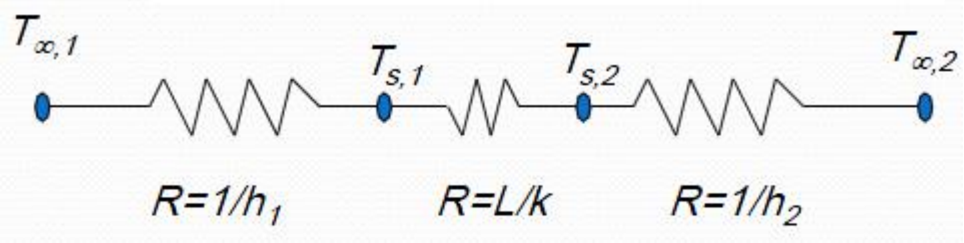


Combined Modes

You must consider the following:
– Convection from hot fluid to wall
– Conduction through wall
– Convection from wall to cold fluid

$$q_x = \frac{T_{\infty,1} - T_{s,1}}{1/h_1 A} = \frac{T_{s,1} - T_{s,2}}{L/kA} = \frac{T_{s,2} - T_{\infty,2}}{1/h_2 A}$$

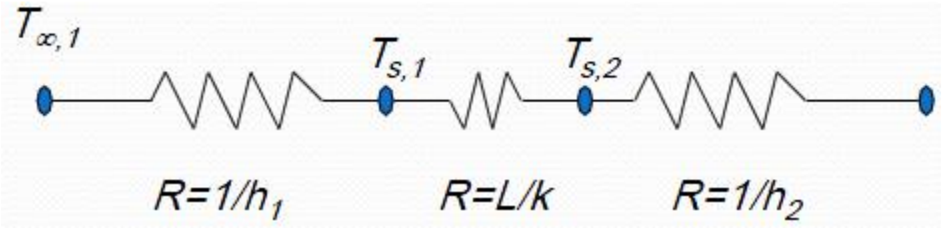




Combined Modes

In terms of overall temperature difference:

$$\boxed{q_x = \frac{T_{\infty,1} - T_{\infty,2}}{R_{tot}}}$$



For a set of resistors in series, the total thermal resistance is:

$$R_{tot} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$$

$$q_x = \frac{T_{\infty,1} - T_{s,1}}{1/h_1 A} \qquad \text{(solve for } T_{s,1}\text{)}$$

$$q_x = \frac{T_{s,2} - T_{\infty,2}}{1/h_2 A} \qquad \text{(solve for } T_{s,2}\text{)}$$

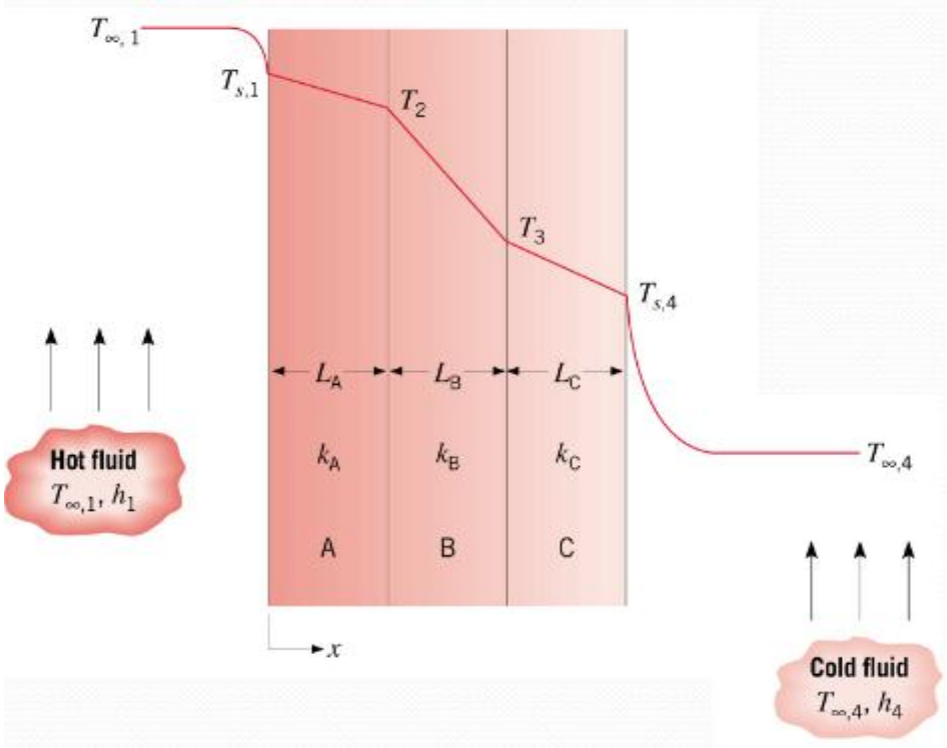
Composite Walls

Composite Walls

Layers of different materials may comprise the wall of the object. These layers may involve any number of series or parallel thermal resistances . We can use "equivalent thermal circuits" to solve these problems. Typically we use this method to determine the overall heat transfer coefficient, U.

Consider the following example:

Express the following geometry in terms of an equivalent thermal circuit ?



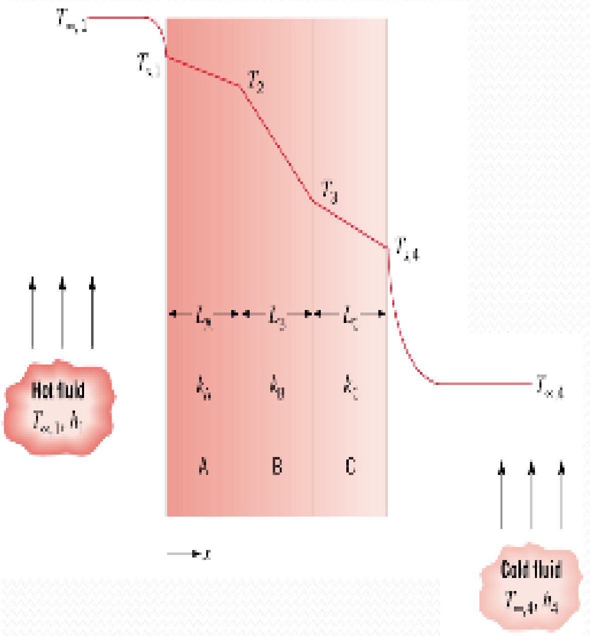
Composite Walls

For engineers, problems like these are solved on a day-by-day basis. For convenience of daily use, we simplify the problem into a overall heat transfer equation.

$$R_{tot} = \frac{1}{A}\left[(\frac{1}{h_1}) + (\frac{L_A}{k_A}) + (\frac{L_B}{k_B}) + (\frac{L_C}{k_C}) + (\frac{1}{h_4}) \right]$$

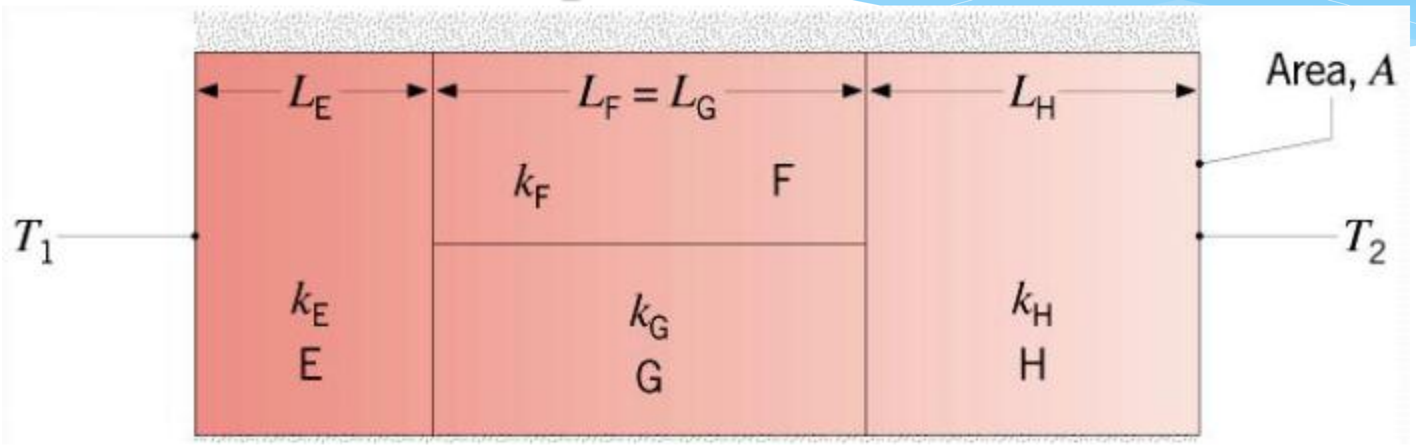
$$U = \frac{1}{R_{tot}A} = \frac{1}{[(1/h_1)+(L_A/k_A)+(L_B/k_B)+(L_C/k_C)+(1/h_4)]}$$

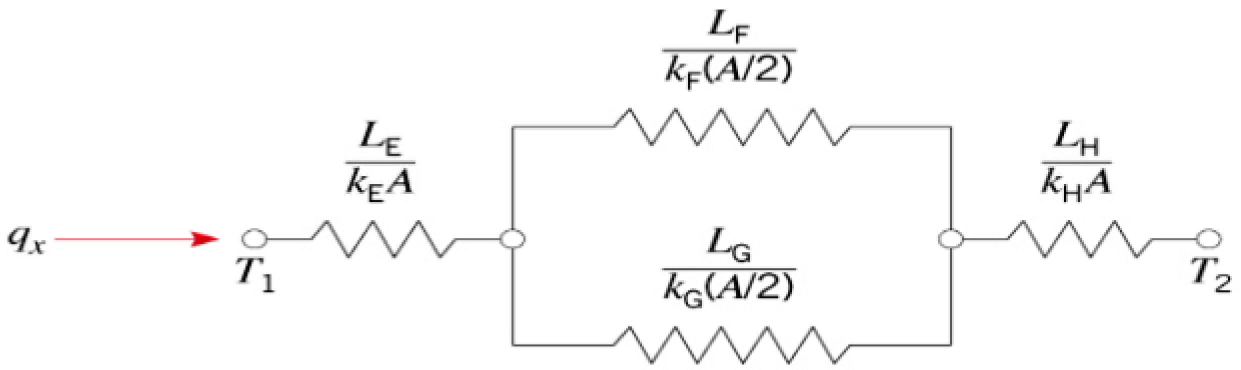
$$\therefore \quad q_x = UA\Delta T$$



The problem given above is an example of a simple circuit. More complex circuits can arise when drawing up equivalent thermal circuits for a series-parallel composite wall.

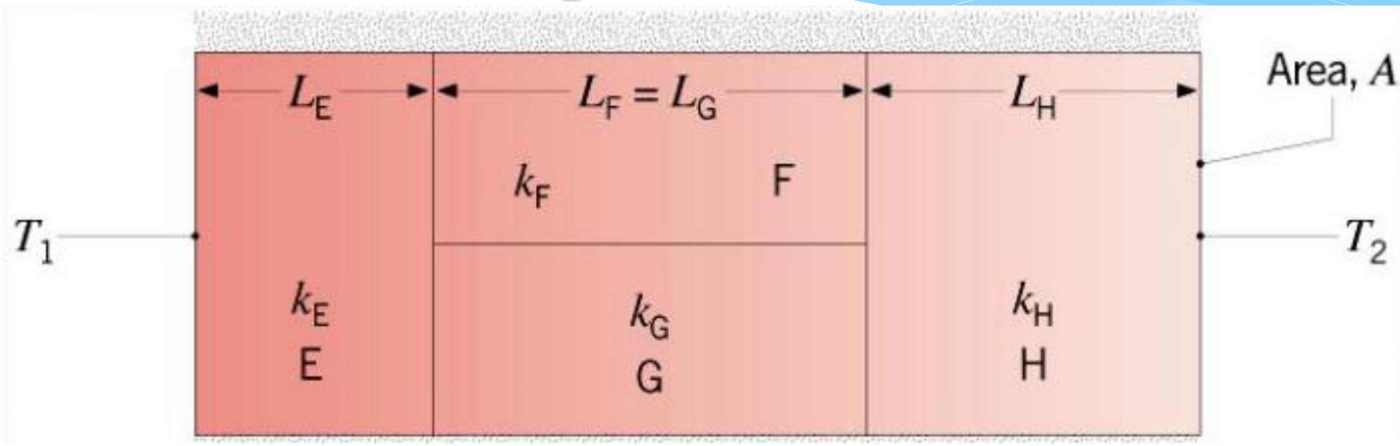
Composite Walls

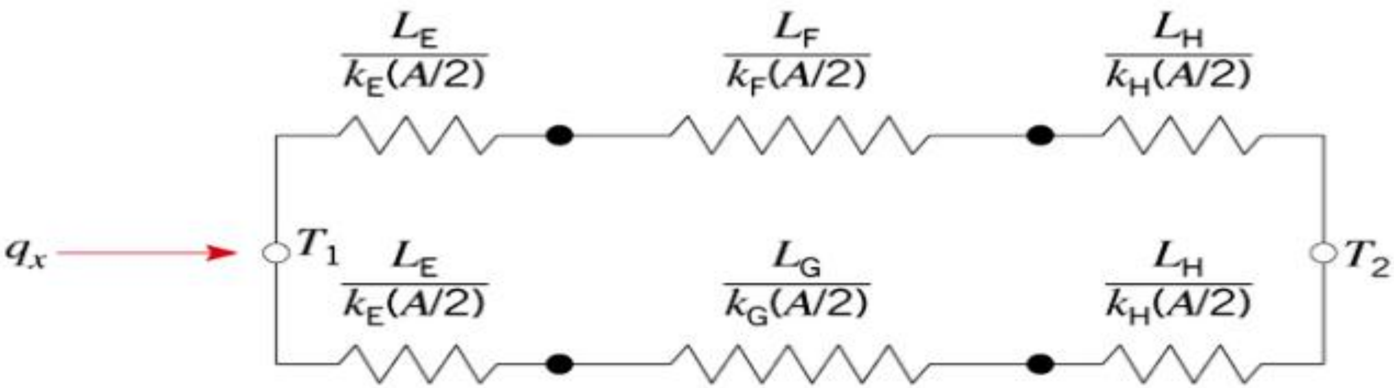




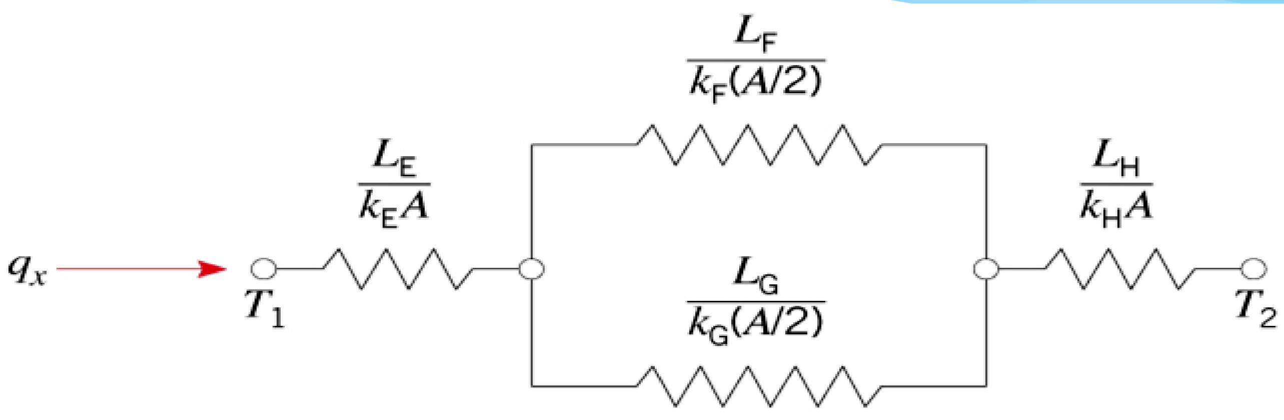
- **For resistances in series:** $R_{tot}=R_1+R_2+\ldots+R_n$
- **For resistances in parallel:** $1/R_{tot}=1/R_1+1/R_2+\ldots+1/R_n$

Composite Walls





Composite Walls

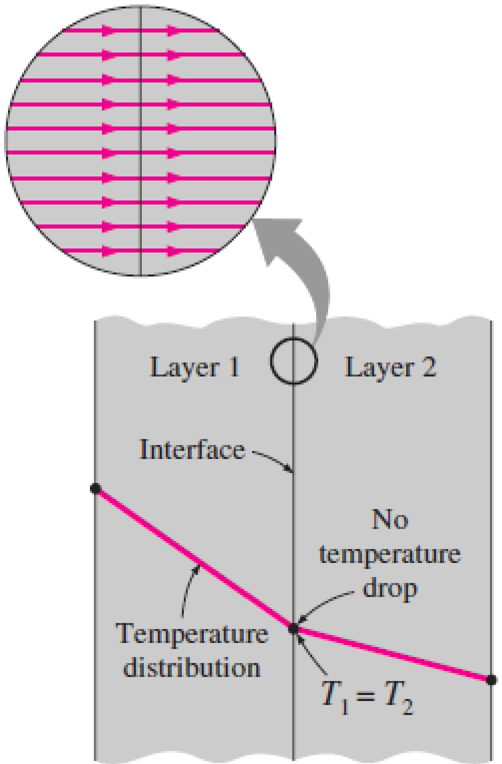


(a)

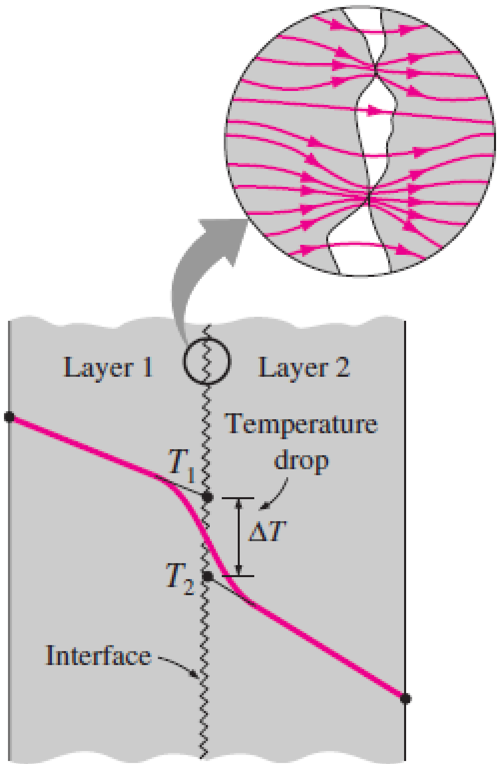
$$\boxed{R_{tot} = R_E + R_{eq} + R_H}$$

$$\boxed{R_{tot} = \frac{L_E}{k_E A} + \left(\frac{1}{\frac{L_F}{k_F(A/2)}} + \frac{1}{\frac{L_G}{k_G(A/2)}} \right)^{-1} + \frac{L_H}{k_H A}}$$

Thermal Contact Resistance



(a) Ideal (perfect) thermal contact



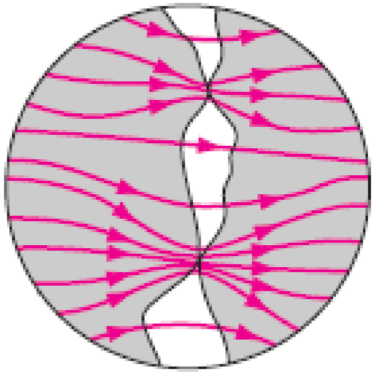
(b) Actual (imperfect) thermal contact

Thermal Contact Resistance

Heat transfer through the interface is the sum of the heat transfers through the solid contact spots and the gaps in the noncontact areas and can be expressed as:

$$Q = Q_{contact} + Q_{gap}$$

$$Q = h_c \, A \, \Delta T_{\text{interface}}$$



Where:

A; is the apparent interface area (which is the same as the cross-sectional area of the rods)

$T_{interface}$; is the effective temperature difference at the interface.

hc; is called the thermal contact conductance which corresponds to the convection heat transfer coefficient.

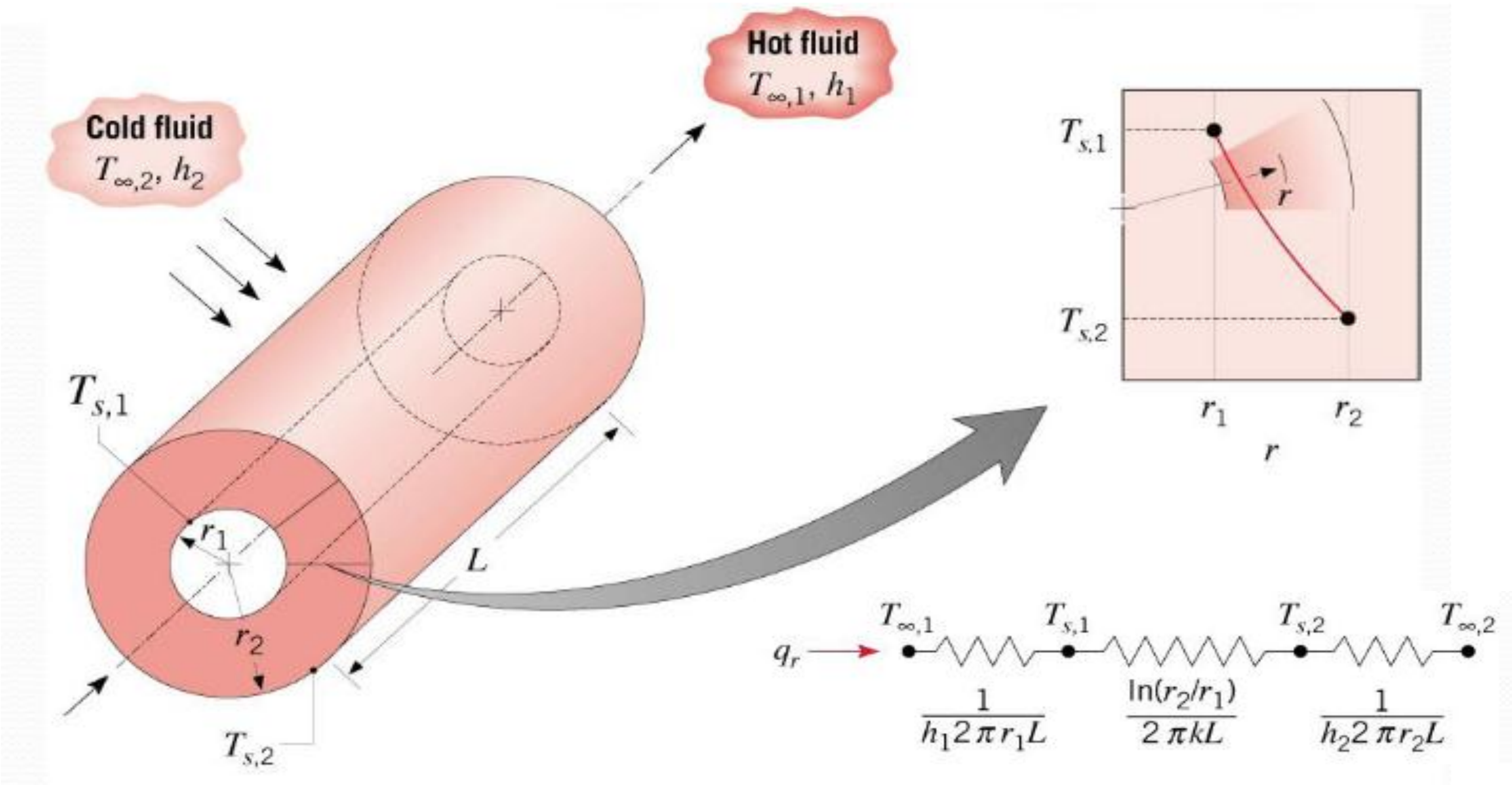
Thermal Contact Resistance

the thermal contact conductance is related to thermal contact resistance by:

$$R_c = \frac{1}{h_c}$$

Radial Combined Modes

Consider a hollow cylinder, whose inner and outer surfaces are exposed to fluids at different temperatures



Radial Combined Modes

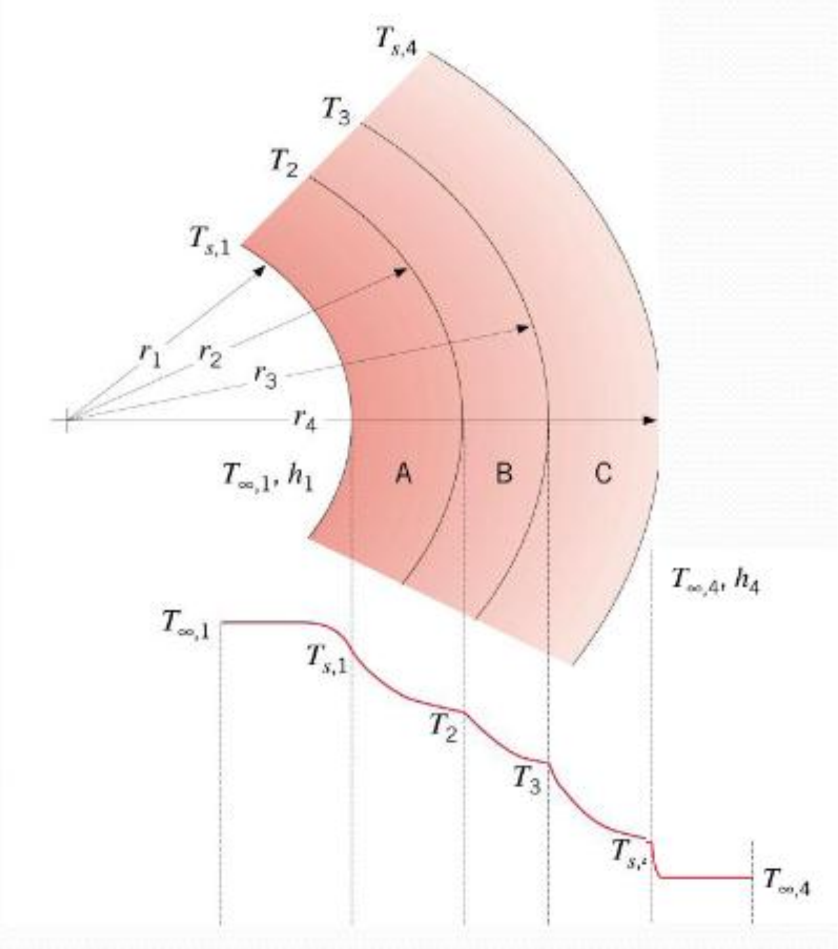
Based on the previous solution, the heat transfer rate can be calculated In terms of equivalent thermal circuit :

$$q_r = \frac{T_{\infty,1} - T_{\infty,2}}{R_{tot}}$$

$$R_{tot} = \frac{1}{h_1(2\pi r_1 L)} + \frac{\ln(r_2/r_1)}{2\pi k L} + \frac{1}{h_2(2\pi r_2 L)}$$

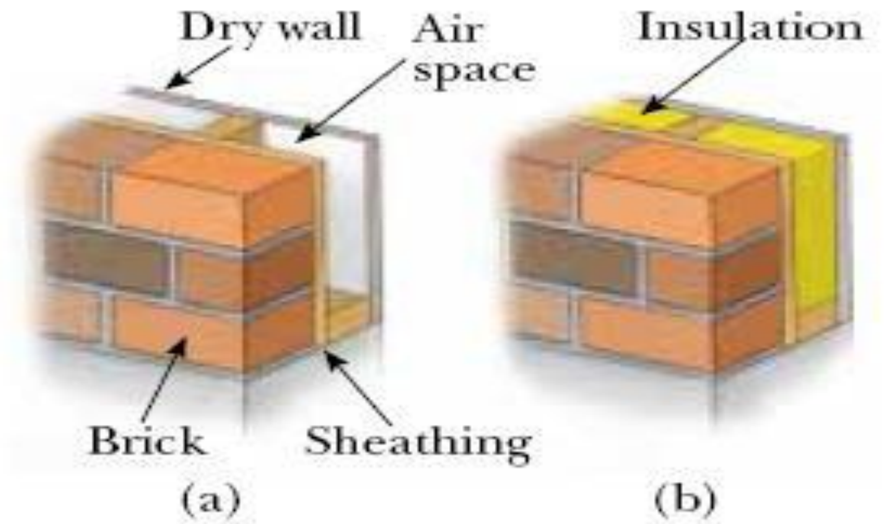
Radial Composite Walls

Express the following geometry in terms of a an equivalent thermal circuit?

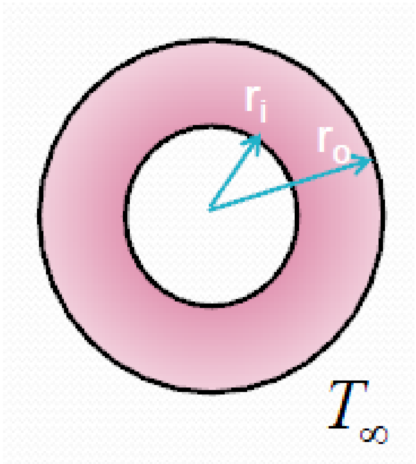


Critical Thickness of Insulation

Thermal insulations consist of low thermal conductivity materials combined to achieve an even lower system thermal conductivity.



An exterior house wall containing
(a) an air space and (b) insulation.



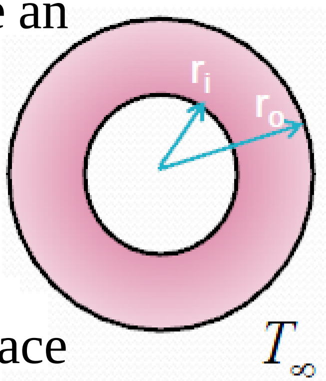
For small diameter tubes it is sometimes possible to increase heat loss by adding insulation on the outer surface!

Critical Thickness of Insulation

Heat loss from a pipe:

$$\dot{q} = hA(T_s - T_\infty)$$

- If A, is increased, q will increase. When insulation is added to a pipe, the outside surface area of the pipe will increase. This would indicate an increased rate of heat transfer



- The insulation material has a low thermal conductivity,
  - it reduces the conductive heat transfer
  - lowers the temperature difference between the outer surface temperature of the insulation and the surrounding bulk fluid temperature.

• This contradiction indicates that there must be a critical thickness of insulation.

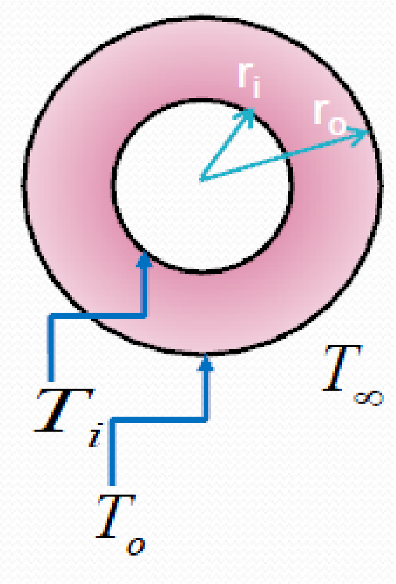
  - The thickness of insulation must be greater than the critical thickness, so that the rate of heat loss is reduced as desired.

Critical Thickness of Insulation

For small diameter tubes it is sometimes possible to increase heat loss by adding insulation on the outer surface!

Conduction through the insulation (cylinder):

$$\boxed{q_r = \frac{T_i - T_o}{\dfrac{\ln(r_o / r_i)}{2\pi kL}} \Rightarrow q_r \frac{\ln(r_o / r_i)}{2\pi kL} = T_i - T_o}$$



Convection from outer surface:

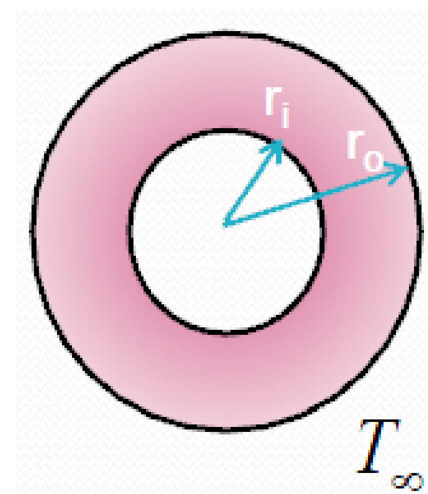
$$T_o - T_\infty = q\frac{1}{h_o(2\pi r_o L)}$$

Combining these two,

$$T_i - T_\infty = q\frac{1}{h_o(2\pi r_o L)} + q\frac{1}{2\pi kL}\ln\left(\frac{r_o}{r_i}\right)$$

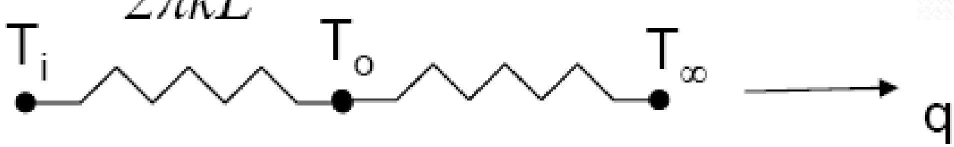
Critical Thickness of Insulation

$$q = \frac{T_i - T_\infty}{\dfrac{\ln\left(\dfrac{r_o}{r_i}\right)}{2\pi kL} + \dfrac{1}{h_o(2\pi r_o L)}}$$



We could have also obtained this equation by using equivalent circuit theory. Check it out!

$$\frac{\ln\left(\dfrac{r_o}{r_i}\right)}{2\pi kL} \qquad \frac{1}{h_o(2\pi r_o L)}$$



Critical Thickness of Insulation

The numerator is a given CONSTANT in the heat flow equation above. To maximize „q", we must minimize the denominator by changing r_o:

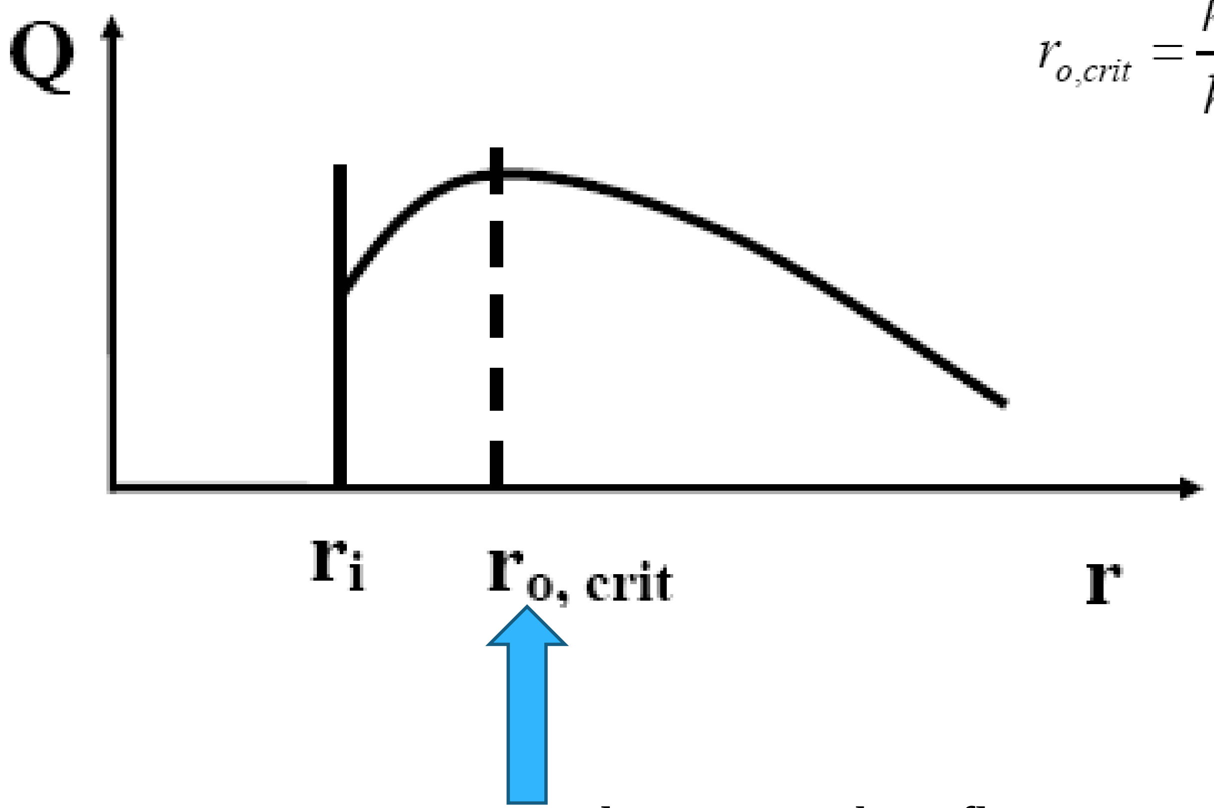
$$\frac{dq}{dr_o} = 0$$

$$\frac{d}{dr_o}\left(\frac{\ln\left(r_o/r_i\right)}{k} + \frac{1}{h_o(r_o)}\right) = 0$$

Therefore,

$$r_{o,crit} = \frac{k}{h}$$

Critical Thickness of Insulation



$$r_{o,crit} = \frac{k}{h}$$

Minimum resistance and maximum heat flow

Summary:

- We obtained temperature distributions, thermal resistances, heat flow, for problems involving steady-state, one-dimensional conduction in Cartesian, cylindrical and spherical coordinates, without energy generation and for different applications